MIND MAP

Sequence & Series

1. Arithmetic Progression (A.P.)

• **General Form of an A.P.**

If 'a' is the first term and 'd' is the common difference, then the standard appearance of an A.P. is

a, a+d, a+2d, a+3d .....

• **General Term of an A.P.**

$T_n = l = n^{th}$ term or last term $\boxed{T_n = a + (n-1)d}$

Note : n^{th} term of an A.P. is of the form $Pn + Q$, i.e. a linear expression in n.

2. Summation of n terms of an A.P.

$\boxed{S_n = \frac{n}{2}[2a + (n-1)d]}$ or $\boxed{S_n = \frac{n}{2}[a + l]}$

Frequently used summations

(1) Sum of first 'n' natural numbers :

$$1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}$$

(2) Sum of first 'n' odd natural numbers :

$$1 + 3 + 5 + \ldots\ldots + (2n-1) = \frac{n(1+2n-1)}{2} = n^2$$

(3) Sum of first 'n' even natural numbers :

$$2 + 4 + 6 + \ldots\ldots + 2n = \frac{n(2+2n)}{2} = n(n+1)$$

Points to Remember

(a) Sum of n terms of an A.P. is of the form $S_n = Pn^2 + Qn$, i.e. quadratic expression in n.

(b) nth term in terms of S_n: $T_n = S_n - S_{n-1}$

3. Properties of an A.P.

Property-1 If a, b, c are in AP, then $\Rightarrow b - a = c - b \Rightarrow 2b = a + c$

Property-2 : 2 , 4, 6 , 8, ... are in A.P.

Add k 2+k , 4+k, 6+k , 8+k, ... are in A.P.
Common Difference : d

2 , 4, 6 , 8, ... are in A.P.

Multiply by k 2.k , 4.k, 6.k , 8.k, ... are in A.P.
Common Difference : kd

2 , 4, 6 , 8, ... are in A.P.

Divide by k 2/k , 4/k, 6/k , 8/k, ... are in A.P.
Common Difference : d/k

Property-3 : In an A.P., summation of k^{th} term from beginning and k^{th} term from the last is always constant which is equal to summation of first and last terms.

$$\boxed{T_k + T_{n-k+1} = \text{constant} = a + \ell}$$

Property-4 : Any term of an AP (except the first & last) is equal to half the sum of terms which are equidistant from it. $T_1, T_2, T_3, \ldots T_{r-k}, \ldots T_r, \ldots T_{r+k}, \ldots$

$$T_r = \frac{T_{r-k} + T_{r+k}}{2}, k < r \quad \text{For } k = 1, T_r = \frac{T_{r-1} + T_{r+1}}{2}$$

Property-5 : If we pick the terms of an A.P. in a particular interval, then picked sequence is also an A.P. with *common difference (d_f) equals to interval times the original common difference (d_i).*

$$\boxed{d_f = (\text{Interval}).d_i}$$

Property-6 : If $a_1, a_2, a_3, \ldots, a_n$ are in A.P. (d_1) and $b_1, b_2, b_3, \ldots$, are also in A.P. (d_2)

Then, $(a_1 + b_1), (a_2 + b_2), (a_3 + b_3), \ldots, (a_n + b_n)$ are in A.P. (c.d. = $d_1 + d_2$)

$(a_1 - b_1), (a_2 - b_2), (a_3 - b_3), \ldots, (a_n - b_n)$ are also in A.P. (c.d. = $d_1 - d_2$)

But, $a_1b_1, a_2b_2, a_3b_3, \ldots, a_nb_n$ *or* $\frac{a_1}{b_1}, \frac{a_2}{b_2}, \frac{a_3}{b_3}, \ldots, \frac{a_n}{b_n}$ may or may not be in A.P.

4. Supposition of terms of an A.P.

(A) If number of terms be ODD, then we take 'a' as middle term and 'd' as its common difference.

3 terms in A.P. as : $a - d, a, a + d$

5 terms in A.P. as : $a - 2d, a - d, a, a + d, a + 2d$

(B) If number of terms be EVEN, then we take 'a - d' & 'a + d' as middle terms and '2d' as its common difference.

4 terms in A.P. as : $a - 3d, a - d, a + d, a + 3d$

6 terms in A.P. as : $a - 5d, a - 3d, a - d, a + d, a + 3d, a + 5d$ & so on.

NOTE: Here, 'a' is not the first term of an A.P.

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5. Arithmetic Mean (A.M.)

All the terms inserted between two given quantities (or numbers) such that the whole sequence thus formed shall be an A.P., then these inserted terms are known as *Arithmetic Means.*

(i) Insertion of an A.M. between two no.s

Let two numbers be 'a' and 'b'
& a , A , b are in A.P.
Where, A = Arithmetic Mean

$$\boxed{A = \frac{a+b}{2}}$$

(ii) Insertion of 'n' A.M.'s between two no.s

$A_1 = a + d$
$A_2 = a + 2d$
$\vdots$
$A_n = a + nd$

$$\boxed{d = \frac{b-a}{n+1}}$$

(iii) Summation of 'n' A.M.'s between two no.s

$$\boxed{\sum_{i=1}^{n} A_i = nA}$$

(iv) Arithmetic Mean (A.M.) of 'n' numbers

Given any 'n' numbers $a_1, a_2, a_3, \ldots, a_n$ then their Arithmetic Mean (A_n) is :

$$\boxed{A_n = \frac{a_1 + a_2 + a_3 + \ldots + a_n}{n}}$$

6. Geometric Progression (G.P.)

In the sequence of non-zero terms, if each term bears the same constant ratio with its immediately preceding term, then that sequence is called as *Geometric Progression* and this constant ratio is called as Common Ratio (r) .

• **General Form of a G.P.**

$a \;\rangle\; ar \;\rangle\; ar^2 \;\rangle\; ar^3 \;\rangle\; \ldots\ldots$

where, first term = 'a' & common ratio = 'r'

• **General Term of a G.P.**

$T_n = l = n^{th}$ term or last term $\boxed{T_n = a(r)^{n-1}}$

• **Common Ratio of a G.P.**

$$r = \frac{T_2}{T_1} = \frac{T_3}{T_2} = \ldots\ldots = \frac{T_n}{T_{n-1}}, \quad \text{where } n \geq 2$$

Note : (a) If $r > 1 \Rightarrow$ G.P. is said to be increasing
(b) If $r \in (0,1) \Rightarrow$ Decreasing G.P.
(c) If $r < 0 \Rightarrow$ Consecutive terms of G.P. are of opposite sign.

• **m^{th} term from the END of a G.P.**

T_m (From the End) $=$ T_{n-m+1} (From the Beginning)

$= a[r]^{(n-m+1)-1} = a[r]^{n-m}$

Short method

G.P.$_1$: $a, ar, ar^2, \ldots\ldots\ldots, ar^{n-2}, ar^{n-1}$

G.P.$_2$: $ar^{n-1}, ar^{n-2}, \ldots\ldots, ar^2, ar, a$

T_m (From the End in G.P.$_1$) $=$ T_m (From the Beginning in G.P.$_2$)

New First term (A) $= ar^{n-1}$

New Common ratio (R) $= \frac{a}{ar} = \frac{1}{r}$

7. Summation of 'n' terms of a G.P.

$$\boxed{S_n = \frac{a(1-r^n)}{1-r} \text{ OR } S_n = \frac{a(r^n-1)}{r-1}, r \neq 1}$$

Sum of infinite terms of a G.P.

$$\boxed{S_\infty = \frac{a}{1-r}; |r| < 1}$$

8. Properties of G.P.

1. If a, b, c are in G.P., then $\frac{b}{a} = \frac{c}{b} \Rightarrow \boxed{b^2 = ac}$
2. If a, b, c,...... are in G.P., then $ka, kb, kc, \ldots$ (where $k \neq 0$) ... are in G.P
3. If a, b, c,...... are in G.P., then $\frac{a}{k}, \frac{b}{k}, \frac{c}{k}, \ldots$ (where $k \neq 0$) ... are in G.P
4. If a, b, c,...... are in G.P., then $a^k, b^k, c^k, \ldots$ are also in G.P

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8. Properties of G.P.

5. In a G.P., product of k^{th} term from beginning and k^{th} term from the last is always constant which is equal to the product of its first and last terms.

$$\boxed{T_k \cdot T_{n-k+1} = \text{constant} = a \cdot \ell}$$

6. In a positive G.P., every term (except first and last terms) is equal to square root of product of its two terms which are equidistant from it.

$$\boxed{T_r = \sqrt{T_{r-k} \cdot T_{r+k}}\,,\ k < r}$$

7. If the terms of a given G.P. are chosen at regular intervals, then the new sequence is also a G.P.

8. If a_1, a_2, a_3 ..... are in G.P & b_1, b_2, b_3, ... are also in G.P with common ratios r_1 and r_2 respectively, then

a_1b_1, a_2b_2, a_3b_3, ... & $\frac{a_1}{b_1}, \frac{a_2}{b_2}, \frac{a_3}{b_3}$......

will also form a G.P. whose common ratio will be r_1r_2 and $\frac{r_1}{r_2}$ respectively.

9. If a, b, c .. is a G.P. of positive terms, then $\log(a), \log(b), \log(c) \ldots$ will be in A.P.

9. Supposition of terms of a G.P.

1. If number of terms be ODD, then we take 'a' as middle term and 'r' as its common ratio.

3 terms in G.P. as : $\frac{a}{r}, a, ar$

5 terms in G.P. as : $\frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2$

2. If number of terms be EVEN, then we take 'a/r' and 'ar' as middle terms and 'r^2' as its common ratio.

4 terms in G.P. as : $\frac{a}{r^3}, \frac{a}{r}, ar, ar^3$

6 terms in G.P. as : $\frac{a}{r^5}, \frac{a}{r^3}, \frac{a}{r}, ar, ar^3, ar^5$

NOTE : Here, 'a' is not the first term of G.P.

9. Geometric Mean

All the terms inserted between two given numbers such that the whole sequence thus formed shall be a G.P., then these inserted terms are called as Geometric Means (GM).

(i) Single G.M. between two numbers

Let 2 positive real no.s be a & b

a, G, b are in G.P., $\boxed{\Rightarrow G^2 = ab}$ $\boxed{\Rightarrow G = \sqrt{ab}}$

(ii) 'n' G.M's between two numbers

a, $G_1, G_2, \ldots, G_n$, b are in G.P.,

Hence, $\boxed{\prod_{i=1}^{n} G_i = \left(\sqrt{ab}\right)^n = G_n}$

(iii) Geometric Mean (G.M.) of 'n' numbers

$$G_n = (g_1 \cdot g_2 \cdot g_3 \ldots\ldots\ldots g_n)^{\frac{1}{n}}$$

9. Harmonic Progression (H.P.)

A non-zero sequence is said to be in H.P. if the reciprocals of its terms are in A.P.

a_1, a_2, a_3, ... are in H.P., if and only if $\frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}$, ... are in A.P.

General form of a H.P.

If 'a' is the first non-zero term 'd' is non-zero common difference of an A.P., then the standard H.P. is

$$\frac{1}{a}, \frac{1}{a+d}, \frac{1}{a+2d}, \ldots\ldots, \frac{1}{a+(n-1)d}$$

Note :

1. To find general term of H.P., convert it into corresponding A.P. and then find its n^{th} term & then reciprocate it.
2. There is no general formula for finding the sum of 'n' terms of H.P.
3. If a, b, c are in H.P. $\Rightarrow \frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P.

$$\Rightarrow b = \frac{2ac}{a+c}$$

10. Harmonic Mean (H.M.)

All the terms inserted between two given numbers such that the whole sequence thus formed shall be a H.P., then these inserted terms are called as Harmonic Mean.

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10. Harmonic Mean (H.M.)

Single H.M. between two numbers

Let two numbers be a & b; a, H, b are in H.P.

$\Rightarrow \frac{1}{a}, \frac{1}{H}, \frac{1}{b}$ are in A.P. $\Rightarrow H = \frac{2ab}{a+b}$

Note : When three quantities are in H.P. then the middle one is called the Harmonic Mean of the other two. i.e., if a, b, c are in H.P. then

$$\boxed{b = \frac{2ac}{a+c}}$$

n - H.M.'s between a and b

Let two positive numbers be 'a' & 'b'

a, H_1, H_2, ..., H_n, b are in H.P.

$\therefore \frac{1}{a}, \frac{1}{H_1}, \frac{1}{H_2}, \ldots\ldots, \frac{1}{H_n}, \frac{1}{b}$ are in A.P

$\Rightarrow \frac{1}{b} = \frac{1}{a} + [(n+2)-1]d$ & $\sum_{i=1}^{n} \frac{1}{H_i} = \frac{n}{H}$

Harmonic Mean of 'n' numbers

Given any 'n' positive real no.s $a_1, a_2, a_3, \ldots\, a_n$ then, its harmonic mean (H_n) is defined by

$$H_n = \frac{n}{\sum_{k=1}^{n} \frac{1}{a_k}} = \frac{n}{\frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3} \ldots \frac{1}{a_n}}$$

11. Rel. Between A.M., G.M. & H.M.

(a) If a and b are two positive numbers A, G, H are their AM, GM & HM respectively,

$A = \frac{a+b}{2} \quad G = \sqrt{ab} \quad H = \frac{2ab}{a+b}$

$\boxed{G^2 = A.H}$ A, G, H are in G.P.

(b) If $a_1, a_2, a_3 \ldots\ldots a_n$ are 'n' positive no.s in G.P. A_n, G_n, H_n are their AM, GM & HM respectively, then A_n, G_n, H_n are in G.P. $\boxed{\Rightarrow G_n^2 = A_n H_n}$

(c) If $a_1, a_2, a_3 \ldots\, a_n$ are 'n' positive no.s, then

$\boxed{A.M \geq G.M \geq H.M}$ Equality holds only when $a_1 = a_2 = a_3 = \ldots = a_n$.

(d) Root Mean Square (RMS) :

Let 2 positive no.s be 'a' and 'b', then

$$\boxed{RMS = \sqrt{\frac{a^2+b^2}{2}}} \quad \boxed{RMS \geq AM \geq GM \geq HM}$$

12. Arithmetico - Geometric Progression (AGP)

General Form of an A.G.P.

a.b , (a + d).br , (a + 2d).br^2, .., $\boxed{[a + (n-1)d].br^{n-1}}$, ...

General Term of A.G.P.

Sum of Infinite Terms of an A.G.P.

$$\boxed{S_\infty = b\left(\frac{a}{1-r} + \frac{dr}{(1-r)^2}\right)}$$

13. Sigma Notations (S)

(a) $\sum_{r=1}^{n} (a_r \pm b_r) = \sum_{r=1}^{n} a_r \pm \sum_{r=1}^{n} b_r$ (b) $\sum_{r=1}^{n} k\, a_r = k \sum_{r=1}^{n} a_r$

(c) $\sum_{r=1}^{n} (a_r . b_r) \neq \sum_{r=1}^{n} a_r . \sum_{r=1}^{n} b_r$ (d) $\sum_{r=1}^{n} \frac{a_r}{b_r} \neq \frac{\sum_{r=1}^{n} a_r}{\sum_{r=1}^{n} b_r}$

Remember (a) $\sum_{r=1}^{n} k = k \sum_{r=1}^{n} 1 = k . n$

(b) Sum of the 1st'n' natural no.s $\sum_{r=1}^{n} r = \frac{n(n+1)}{2}$

(c) Sum of the squares of the 1st 'n' natural no.s

$$\sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}$$

(d) Sum of the cubes of the 1st 'n' natural no.s

$$\sum_{r=1}^{n} r^3 = \left(\sum_{r=1}^{n} n\right)^2 = \left(\frac{n(n+1)}{2}\right)^2$$

(e) Sum of 1st 'n' odd natural no.s $= \sum_{r=1}^{n} (2r-1) = n^2$

(f) Sum of 1st 'n' even natural no.s $= \sum_{r=1}^{n} (2r) = n(n+1)$